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HEAT AND WATER VAPOR EXCHANGE BETWEEN WATER SURFACE AND ATMOSPHERE

Wilfried Brutsaert

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HEAT AND WATER VAPOR EXCHANGE BETWEEN WATER SURFACE AND ATMOSPHERE

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Project 16130 DIP
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ABSTRACT

The physical and mathematical aspects of simultaneous turbulent heat and water vapor exchange between a large open water body and the surrounding atmosphere were studied. Thus analytical and numerical solutions were developed for various conditions of fetch, surface roughness, atmospheric stability, etc., that are likely to be of physical importance.

One of the main findings was that in spite of some theoretical limitations the semi-empirical turbulent diffusion model provides a method for the prediction of heat and water vapor transfer, that should be useful for engineering calculations. Although the interaction between momentum, sensible heat and water vapor is considerable, for evaporation or cooling calculation purposes it is probably permissible to uncouple their transfer mechanisms provided the water surface temperature is known and the averaging period under consideration is of the order of, say, a week.

In addition the validity of the semi-empirical assumption in the near-water surface layer was analyzed by determining the anisotropy of the eddy diffusivity, the effects of radiative transfer and of water wave action on the eddy diffusivity. Finally, a practical method was developed to determine evapotranspiration from the surrounding land surface based on the geostrophic drag concept.

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CONTENTS

Section		Page
I	Conclusions	1
II	Recommendations	3
III	Introduction	5
IV	Turbulent Diffusion Model	7
V	Discussion	11
VI	Prediction of Evaporation from a Large Water Body	21
VII	Acknowledgements	25
VIII	References	27
IX	Publications	29
X	Appendix	31

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SECTION I

CONCLUSIONS

1. The semi-empirical turbulent diffusion model with Reynolds' analogy provides a method for the prediction of heat and water vapor transfer from a water surface which should be adequate for engineering calculations.

2. For practical evaporation and or sensible cooling calculations with averaging periods of, say, one week, or more, it is permissible to uncouple momentum, heat and vapor transfer. For "average" conditions, namely a near-neutral atmosphere and a roughness of the water surface of approximately 0.02 cm, the theoretical calculations are in agreement with the empirical formula given by equations (16) and (17).

3. The eddy diffusivity used in the semi-empirical approach is highly anisotropic. However, for calculations of the fluxes near the water surface only the vertical component has to be included. The effect of the longitudinal, lateral and off-diagonal components is for all practical purposes negligible.

4. With proper values of the parameters the power law for the wind profile in the atmospheric surface layer gives good results in the prediction of turbulent transfer. The calculations are in agreement with solutions obtainable by means of the more complicated laws such as the logarithmic, log-linear and the other generally accepted profile laws.

5. The effect of the difference in surface roughness of the water and the surrounding land on heat and vapor transfer near the surface is very small, since the air flow and the surface shear stress adjust rather quickly to the roughness of the new surface. Because presently available methods are not quite adequate to predict flow in the higher regions of the boundary layer, this points to the importance of a better understanding of the transfer mechanisms right at the surface.

6. The discontinuity of surface temperature of the evaporating water body and the surrounding land has a strong effect on the turbulent fluxes. For example, a relatively cold and neutral air moving over a relatively warmer water surface may experience extreme instabilities.

7. Under conditions normally encountered in the lower atmosphere the effect of molecular and radiative transport mechanisms and of water vapor stratification on turbulent transfer is not always negligible. Inclusion of these effects in the semi-empirical turbulence theory produces expressions for the wind velocity profile and other turbulence parameters which are in good agreement with available experimental data.

8. The turbulent transfer over a water surface disturbed by waves is affected not only by atmospheric conditions but also by sea-state. Except under conditions of developing swell, this effect is a function of c/u_*, where c is the phase velocity of the dominant wave and u_* the friction velocity.

9. Evapotranspiration from large areas can be determined satisfactorily by means of the geostrophic drag concept. The method can be applied by using standard meteorological data obtained at the surface and in the atmospheric boundary layer.

SECTION II

RECOMMENDATIONS

The recommendations of the project are best formulated by summarizing areas of research which in the light of this study appear to require further work. The findings of the present study allow the prediction of heat and water vapor exchange for periods larger than one week. However, for more accurate short term calculations the following topics should be studied in greater detail.

1. The transfer mechanism in the air right at an air-wet surface interface. This involves a better definition of the flow, the nature of the so-called laminar sublayer and the turbulence at the interface; also, a determination of the relationship between the flux of heat and water vapor from the surface into the atmosphere and such factors as the roughness of the surface, the viscosity of the air, the molecular diffusivities of the diffusing substances and the shear stress of the wind.

2. The study of simultaneous heat and water vapor transfer for an inversion, that is under extremely stable conditions in the neighborhood of the critical Richardson number. The nature of this type of phenomenon dictates that it be considered under unsteady conditions, and that radiative transfer be included in the general formulation of the problem.

3. The inclusion of radiation in the formulation of the transfer mechanisms in the atmospheric surface layer. The equations for radiative transfer are, however, so complicated that this may necessitate the development of a simpler semi-empirical theory analogous to that used to obtain engineerng solutions to turbulence problems.

4. The effect of water waves on the turbulent transfer near the surface. Specifically, this will encompass a better characterization or parameterization of the waves, the definition of the surface roughness in terms of wave field parameters, and the development of similarity functions to describe the turbulence under conditions of developing swell preferably on the basis of a suitable wind wave generating theory.

SECTION III

INTRODUCTION

Large inland water bodies, lakes or manmade reservoirs constitute a valuable resource available to human society. To preserve this natural resource and to allow its optimum utilization a better understanding of the physical properties of these large water bodies and of their interaction with the environment is required. Of critical importance in this respect is the heat and mass exchange at the interface between the water and the surrounding atmosphere.

Large reservoirs, lakes and the continental shelf have also become increasingly attractive as sites for construction of power generating plants and related industries because of the abundance of condenser cooling water. The advent of nuclear power and the rapid increase of such plants make the need for cooling water especially severe and critical. Moreover, the fact that water entering the evaporation process becomes nonrecoverable and is thus a "consumptive" use is fundamental to water resource planning and management. Mass transfer and evaporation must be predicted as accurately as possible since such data are indispensable to design the capacity of manmade reservoirs or to assess the value of natural water bodies for municipal and industrial water supply, navigation, irrigation of agricultural lands and recreation.

Therefore the overall objective of this project was to gain a better understanding of the simultaneous turbulent exchange of momentum, heat and water vapor between a large water body and the surrounding atmosphere for given conditions of local atmospheric advection and of solar and other energy inputs into the water body. The results of the theoretical analysis should allow a better evaluation of some of the practical methods presently available to predict heat transfer and evaporation which are based on simpler but cruder models. A by-product of this investigation is a clearer statement of some of the conditions at the water surface which are essential in the solution of lake circulation and stratification problems. Finally, it should be possible to apply some of the findings of the project in the solution of certain aspects of air pollution problems.

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This objective was accomplished by the analysis of some of the turbulent diffusion mechanisms in the surface layer of the atmosphere. The basic theoretical framework of this analysis is outlined in the next Section.

SECTION IV

TURBULENT DIFFUSION MODEL

A conservative substance admixed in a fluid is transferred relative to a fixed coordinate system first through convective flow of the fluid itself and second through molecular motion relative to the convective motion of the fluid. Under fully turbulent flow conditions the molecular motion may be neglected as compared to the motion resulting from the fluid convection. Decomposing the velocity field and the concentration of a scalar admixture in a time mean and a fluctuation, one can write the equation of continuity for the admixture as follows

$$\frac{dC}{dt} = -\left[\frac{\partial}{\partial x}(\overline{uc}) + \frac{\partial}{\partial y}(\overline{vc}) + \frac{\partial}{\partial z}(\overline{wc})\right] \qquad (1)$$

where C and c are the mean and fluctuating concentration of the admixture, respectively. Similarly, U, V and W are the mean velocity components and u, v and w are the fluctuating velocity components in the mean wind direction x, in the lateral y and in the vertical z, respectively.

It is very difficult to solve equations such as (1) because of the problem of closure. This means that for any problem of turbulent transfer there are always more unknowns than equations. It is for this reason that the semi-empirical theory of turbulence was developed. The fundamental assumption of this approach is that the turbulent flux components $(\overline{uc})$, $(\overline{vc})$ and $(\overline{wc})$ can be related linearly to the mean concentration gradients. For the present problem of diffusion from a water surface at ground level it is permissible to assume that the mean lateral velocity is zero, $V = 0$, and that the mean vertical velocity W is at least an order of magnitude smaller than the horizontal velocity U. Thus equation (1) can be reduced to a relatively simple partial differential equation in C only, (e.g. Brutsaert, 1970)

$$\frac{dC}{dt} = \frac{\partial}{\partial x}\left(K_{xx}\frac{\partial C}{\partial x} + K_{xz}\frac{\partial C}{\partial z}\right)$$

$$+ \frac{\partial}{\partial y}\left(K_{yy}\frac{\partial C}{\partial y}\right) + \frac{\partial}{\partial z}\left(\left(K_{xz}\frac{\partial C}{\partial x} + K_{zz}\frac{\partial C}{\partial z}\right)\right) \qquad (2)$$

where $(d/dt) = U(\partial/\partial x) + W(\partial/\partial z) + (\partial/\partial t)$ and where the K-terms are the components of the eddy diffusivity tensor for the admixture under consideration.

In the case of heat and vapor transfer the conservation equations may be readily obtained merely by replacing C by $c_p T$ and q, respectively, where c_p is the specific heat at constant pressure, T the absolute temperature and q the specific humidity.

Conservation of momentum is formally describable by the Reynolds equations. However, in the atmospheric surface layer the flow is governed mainly by frictional forces due to the proximity of the earth's surface. Also, the mean flow is practically (but not quite) plan-parallel, i.e. as mentioned $U >> W$. All this means that for the present problem it is permissible to consider the mean horizontal wind as a scalar so that (2) also describes the conservation of momentum if C is replaced by U.

Thus simultaneous turbulent momentum, heat and water vapor diffusion near a water surface is governed by the equation of continuity

$$\frac{\partial U}{\partial x} + \frac{\partial W}{\partial z} = 0 \tag{3}$$

and by the equations of conservation of horizontal momentum, sensible heat and water vapor, namely

$$\frac{dU}{dt} = \frac{\partial}{\partial x}\left(K^m_{xx}\frac{\partial U}{\partial x} + K^m_{xz}\frac{\partial U}{\partial z}\right) + \frac{\partial}{\partial y}\left(K^m_{yy}\frac{\partial U}{\partial y}\right) + \frac{\partial}{\partial z}\left(K^m_{xz}\frac{\partial U}{\partial x} + K^m_{zz}\frac{\partial U}{\partial z}\right) \tag{4}$$

$$\frac{dT}{dt} = \frac{\partial}{\partial x}\left(K^h_{xx}\frac{\partial T}{\partial x} + K^h_{xz}\frac{\partial T}{\partial z}\right) + \frac{\partial}{\partial y}\left(K^h_{yy}\frac{\partial T}{\partial y}\right) + \frac{\partial}{\partial z}\left(K^h_{xz}\frac{\partial T}{\partial x} + K^h_{zz}\frac{\partial T}{\partial z}\right) \tag{5}$$

$$\frac{dq}{dt} = \frac{\partial}{\partial x}\left(K^v_{xx}\frac{\partial q}{\partial x} + K^v_{xz}\frac{\partial q}{\partial z}\right)$$

$$+ \frac{\partial}{\partial y}\left(K^v_{yy}\frac{\partial q}{\partial y}\right) + \frac{\partial}{\partial z}\left(K^v_{xz}\frac{\partial q}{\partial x} + K^v_{zz}\frac{\partial q}{\partial z}\right) \qquad (6)$$

where the superscripts m, h and v refer to momentum, heat and water vapor, respectively.

In general, these four equations must be solved simultaneously, subject to, among others, the following boundary conditions

$$U = W = 0 \quad \text{at } z = 0, \text{ all x and y.} \qquad (7)$$

$$L_e F_{VO} + F_{HO} + R_{LU} = (1-\gamma_s)R_S + (1-\gamma_l)R_{LD} + F_W \qquad (8)$$

$$\text{at } z = 0 \text{ and x and y on the wet surface}$$

in which L_e is the latent heat of evaporation, F_{VO} and F_{HO} the vertical fluxes of water vapor and sensible heat, respectively, R_{LU} and R_{LD} the up- and downward long wave radiation fluxes, respectively, R_S the down coming short wave radiation, F_W the heat flux released from below the surface and γ_s and γ_l the albedo's for short and long wave radiation, respectively.

Clearly, the complete mathematical solution of this problem, although not presenting any insurmountable difficulties, is quite involved and harldy practical for design calculations of heat transfer and evaporation. Therefore, the major effort in this project consisted of determining the relative importance of the various terms of equation (2) and the sensitivity of the solutions to different boundary conditions and also of analyzing the degree of interaction among the transfer of heat, water vapor and momentum. This was done primarily to develop and evaluate approximate but simpler methods. In addition, however, because of the uncertainties of certain assumptions concerning the various physical mechanisms, a large amount of attention was also devoted to a better formulation of turbulent and other transfer phenomena.

SECTION V

DISCUSSION

In this section a description and a discussion is given of the studies that were carried out with the basic turbulent diffusion model presented in Section IV.

Relative Importance of Individual Terms in Equation (2). Under certain conditions, which are not uncommon on a long term basis, it is permissible to assume, first, that momentum transfer is strictly vertical, so that equation (4) can be replaced by a one-dimensional wind profile equation with constant roughness and atmospheric stability over the land and water surface with $U = U(z)$ and $W = 0$; and second, that the temperature of the water surface is constant, so that the heat and vapor fields are separable and each can be analyzed by itself. Although this special situation occurs only rarely at any given instant, it is of interest because it allows the determination of the relative importance of the individual terms in the turbulent diffusion equation (2) without the need to consider the complicated interaction among equations (4), (5) and (6).

For this purpose several methods of solution were devised for the turbulent diffusion equation (2) subject to a steady concentration boundary condition at $z = 0$.

First a numerical procedure was developed for the case of a steady velocity profile field under near neutral conditions, which is homogeneous in the horizontal. In the finite difference formulation of this type of equation two difficulties arise caused by the automatic satisfaction of one of the boundary conditions at the surface and by the infinite size of the solution domain. The numerical scheme was designed to overcome these difficulties by the use of appropriate transformations. The results of numerical experiments showed that the effect of the longitudinal diffusion term and of the off-diagonal term are both negative but usually negligible for evaporating or cooling surfaces with an area of at least a few square meters. It was also found that with suitable parameters for roughness and stability the power law can be as useful as the more complicated logarithmic law. The power law was determined to be

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$$U = (5.5/7m)\ u_* \ (z/z_o)^m \qquad (9)$$

where m is close to (1/7), u_* the friction velocity and z_o the roughness length. The details of this numerical study are presented in publications 1 and 2 (Section IX).

A diagonalized form of equation (2) without the K_{xz} terms was also solved analytically for the same type of boundary conditions by means of a regular perturbation method with a small parameter. This solution confirmed essentially the conclusions of the numerical study since it showed that the effect of longitudinal diffusion is negative but negligible for evaporating surfaces larger than say 100 cm in the direction of the wind. The effect of the lateral diffusivity K_{yy} had already earlier been shown to be negligible. This means then that for the problem of evaporation or heat transfer from a surface at ground level equation (2) (and thus equations (4), (5) and (6) as well) may be simplified to

$$\frac{dC}{dt} = \frac{\partial}{\partial z} \left(K \frac{\partial C}{\partial z}\right) \qquad (10)$$

in which the subscripts zz have been dropped from the diffusivity. The details of the perturbation analysis have been presented in publication 3.

Comparison of Solution of Equation (10) with Empirical Data. Several empirical mass transfer formulae are available in the literature for the determination of evaporation from lakes and from pans at ground level. These were compared with the theoretical solution of equation (10) for steady conditions obtained by assuming the validity of Reynolds' analogy and of the power law of equation (9) for the wind profile. It was found that although this theoretical solution, which was first given by Sutton, has its limitations, it is a useful tool for the study of vapor or heat flux under conditions of lateral advection. Conversely the empirical formulae, in particular that proposed by Harbeck, should be quite adequate for many design problems since their general functional form is compatible with this simple theoretical model. For large water surfaces the wind profile power of equation (9) appears to be m = 1/8, and the roughness close to z_o = 0.02 cm. The details of this analysis are described in publication 4.

Interaction between heat and vapor in forced convection. After it had been determined that equation (10) with the assumption of equation (9) can provide solutions in agreement with experimental data, by analogy with (10) the following simplified forms of equations (5) and (6), were considered

$$U \frac{\partial T}{\partial x} = \frac{\partial}{\partial z} \left(K \frac{\partial T}{\partial z} \right) \tag{11}$$

$$U \frac{\partial q}{\partial x} = \frac{\partial}{\partial z} \left(K \frac{\partial q}{\partial z} \right) \tag{12}$$

in which again equation (9) for U was assumed to replace (4) and the Reynolds analogy was assumed valid. The link between these two equations was assumed to be provided by the boundary condition (7) whereas the other boundary conditions were taken as those used in the Sutton solution. The fact that equations (11) and (12) are linked leads naturally to the necessity of solving them simultaneously. The solution was obtained analytically by the construction of Green's function which, when incorporated in the boundary condtions, produces two integral equations. These in turn were solved by transformation into two algebraic equations by means of the Laplace Transformation. The results of this solution showed, how for a simple case sensible heat and water vapor transfer and also the water surface temperature may be expected to depend on the meteorological conditions and on the rate of change of energy content of the water body and the surrounding land. Due to advection the water surface temperature and the turbulent fluxes vary in the downwind direction. In contrast to what is sometimes assumed for a deep lake the surface temperature is usually quite different from the wet-bulb temperature. The main conclusion of this study was, that the error introduced by the use of an experimentally obtained average temperature, in the calculation of evaporation and turbulent sensible heat transfer, is probably quite small, and negligible for practical purposes. Of course, this provides one more indication of the validity of Sutton's solution and Harbeck's formula in situations where horizontal momentum advection is negligible. The details of this interaction analysis are given in publication 5.

Investigation of Various Semi-Empirical Formulations of the Turbulence. Because the validity of the theoretical calculations carried out by means of the solution of equations (4), (5) and (6) is so critically dependent on, first, the mathematical suitability, and second the corespondence to real physical mechanisms of the semi-empirical turbulent diffusion models under various conditions, this aspect of the problem also had to be investigated.

In the solutions described above intensive use was made of the power law for the wind profile as given by equation (9). Actually, this law has been widely used in the solution of numerous turbulent diffusion problems. However, because it is admittedly only approximate and without much of a theoretical model to justify it, such solutions have not always received the attention they deserve among atmospheric scientists. Nevertheless, as pointed out above, evaporation under neutral conditions can be calculated by means of the power law equally well as by means of the logarithmic law. Therefore it was felt desirable, that the power law also be made applicable under non-neutral conditions. This was accomplished by determining C and m in equation (9) as functions of the roughness and of the (Monin-) Obukhov stability length L, through comparison with the log-linear law for stable conditions, and Obukhov's formula (KEYPS) for unstable conditions. As was to be expected, for neutral conditions the results show that m and C are approximately equal to 1/7 and 6, respectively. C increases (decreases) slightly whereas m decreases (increases) with decreasing (increasing) stability. The results are tabulated in publication 6.

The next step in this research consisted of evaluating the effect of certain important factors on the wind profile that have usually been neglected in the past, namely radiative and latent heat transfer, the molecular viscosity of the air and the molecular diffusivities of heat and water vapor in the air. The basic theoretical approach was analogous to Obukhov's. However, the (Monin-) Obukhov stability length was adjusted to obtain a more nearly height-independent similarity. Even though the Obukhov model contains some strong assumptions, the inclusion of the effects of radiation and of the molecular diffusivities on the dynamics of the flow yields a profile law, which in contrast to other previously proposed laws is valid over the whole range of atmospheric stability conditions over

rough as well as over smooth surfaces. For example, the resulting solution appears to agree well with Obuknov's (KEYPS) profile for unstable conditions or neutral conditions over a rough wall or over a smooth wall at some distance from the wall. Very close to the smooth wall, where viscosity is taken into account, the resulting solution reduces to the linear profile of the laminar sublayer. Hence it does not have a singularity right at $z = 0$ for a smooth surface with $z_o = 0$, and it is in good agreement with the generally accepted concept of the "law of the wall". Under stable conditions the resulting solution is in good agreement with the experimentally verified log-linear law. This seems to indicate, that under unstable conditions the effects of radiation and of the molecular viscosity and diffusivities tend to cancel each other and that very little error is introduced if both are neglected. However, under stable conditions these effects probably reinforce each other and must be considered. This new formulation of the eddy diffusivities and the wind profile should be especially useful in turbulent transfer calculations and numerical modeling of the atmospheric surface layer on digital computer. The initial concept of introducing the molecular viscosity into the mixing length under neutral conditions is described in publication 7 and 8.

The project dealt primarily with turbulence as an efficient mechanism of heat and vapor transport. Because radiative transfer has for some time been known to play an important role in the maintenance of turbulence under inversion conditions very close to the critical Richardson number this matter needed further investigation. Thus an expression was derived relating the critical flux Richardson number with the critical (gradient) Richardson number. In contrast to an earlier analysis by Townsend, which was restricted to the atmosphere well outside the earth's boundary layer, the treatment of the problem was applied specifically to turbulent transfer near the surface and it took account of the effect of evaporation on the stability. The effect of radiation on the rate of destruction of the mean square of the temperature was obtained by considering the radiative flux divergence in a stratified atmosphere and by using a simple functional relationship to represent empirical emissivity data. It was found that under conditions normally encountered in the lower atmosphere, the effect of evaporation and radiation on turbulence is not negligible. The critical Richardson number, which is the criterion for the existence or cessation of turbulence, is sensitive to

both factors. There is no definite critical Richardson number but it falls in a range between 0.25, below which turbulence is highly probable, and somewhat larger than 0.5, above which turbulence is unlikely, This critical Richardson number can be expressed in terms of evaporation, radiation and the ratio (σ_w/u_*) of the variance of the vertical velocity fluctuations and the friction velocity; this ratio, (σ_w/u_*) in turn, also appears not to have a definite value. Evaporation and radiation cause it to be larger than unity under neutral conditions. These results show that the proposed model for turbulence and radiation together with the assumption $K^h = K^m$ is consistent with, or at least not contradicted by, the available experimental evidence. However, more research is needed in this area. The details of this analysis have been given in publication 9.

Finally, there has been increasing evidence of the fact that there exists a strong interaction between the turbulent flow of the air and motion of the underlying water waves. Because this interaction is a highly nonlinear process with a complicated feedback mechanism it is still not very well understood. Therefore, first a review was made of recently published data. It appeared that Volkov's measurements taken on the Western Mediterranean were especially suitable. By assuming a similarity function suggested by Kitaygorodskiy a relationship was derived between the actual eddy diffusivity K^m and the apparent eddy diffusivity, K_D^m, estimated purely from the wind velocity profile. This was done in terms of (c/u_*) where c is the phase velocity of the dominant wave and u_* the friction velocity. It was found that the eddy diffusivity near the air-water interface, determined in the usual way from the mean velocity profile, may lead to considerable error depending on the sea state. Thus measurements of the mean wind velocity or humidity profiles lose a great deal of their usefulness unless additional measurements are made of the sea-state. At any rate, a knowledge of the similarity function allows a correction to be applied to the turbulent flux obtained by means of the bulk aerodynamic method on the basis of profile measurements. This matter is described in detail in publication 10. In publication 11, a theoretical derivation is presented of this similarity function for the wind profile gradient and for the standard deviations of the velocity fluctuations. The model is mainly based on the assumption that the relative speed of the air with respect to the celerity of the waves is the most important factor governing the

magnitudes of the wave induced velocity fluctuations. Additional assumptions are that the mean wind profile is logarithmic and that the Reynolds stresses are constant in the vertical. The results were in satisfactory agreement with available experimental data for the case of decreasing swell. For developing swell further research will be required.

The Computation of Evapotranspiration from Large Land Areas. In the determination of heat and vapor losses from a lake or reservoir it is desirable that the evapotranspiration from the surrounding land be known. Most methods, that are available at present, and that are sufficiently practical to be applied with normally observed meteorological data, are almost purely empirical without a firm physical base and quite limited because they were developed for special hydrologic conditions. There are equations that are based on a more rational approach, but they provide only the vertical vapor flux at a given point where they require a detailed knowledge of the energetics and of the turbulence structure of the atmosphere.

Therefore, a practical, yet rational procedure was developed for computing regional evapotranspiration. The assumptions underlying the model are essentially those used in the one-dimensional mass transfer methods, but the shear stress is determined by means of the equation $\tau_o = \rho C_g V_{go}$ in which ρ is the density of the air, C_g is the geostrophic drag coefficient as first proposed by Lettau and V_{go} the surface geostrophic wind. The main advantage of the method is that all data needed in the computation, namely the surface temperature, the pressure of the air, the specific humidity at two levels in the atmospheric boundary layer and the surface geostrophic wind speed can be obtained from standard meteorological data such as Climatological Data, Daily Weather Maps and Rawinsonde Data which are published regularly. Generally, good agreement was obtained between mean monthly evapotranspiration computed for a number of stations and the corresponding evaporation data obtained from Class-A pans. Although this testing is obviously not conclusive, because of the limitations inherent in pan evaporation to determine actual regional evapotranspiration, it does offer some encouraging evidence of the soundness of the model. The details of this study, with a step by step procedure, are given in publication 12.

Heat and Water Vapor Exchange Under Conditions of Unstable Atmospheric Stability and Momentum Advection. In the study of the interaction between heat and water vapor in forced convection described above (reference 5), it was assumed that the momentum transfer is strictly vertical. Therefore, equation (4) could be replaced by a wind velocity profile law $U = U(z)$, $W = 0$, and equations (5) and (6) could be simplified to (11) and (12). Actually in this earlier study a phenomenon, in which as a result of the sudden temperature jump at $x = 0$, the stability of the air would definitely vary in the x-direction, was treated as one in which the stability would affect the flow only in an average way. On a long term basis, as the agreement with the experimental data shows, the effect of this temperature disontinuity on the advection of momentum is probably negligible. However to determine the effect of this discontinuity under extreme conditions it was decided to consider the case in which the air moves from over a relatively cool land surface onto a warmer water surface. This situation, which may create severe instability in the air is not uncommon, for example, over a deep lake in the fall and early winter months.

In recent years the problem of momentum advection in the atmospheric surface layer has received considerable attention. A very thorough literature survey on the problem of a sudden (step-) change of the surface roughness on the air flow under neutral conditions revealed that very close to the surface the flow adjusts rather quickly to the new surface, but that it is still difficult to predict the flow in the higher regions of the boundary layer. This means that a thorough understanding of the transport mechanisms in the immediate neighborhood of the surface is of utmost importance in the determination of the vertical transfer rates. Although there are several methods for analyzing the change in roughness problem, the method consisting of solving equation (4) but with only the vertical diffusion term, namely

$$U \frac{\partial U}{\partial x} + W \frac{\partial U}{\partial z} = \frac{\partial}{\partial z} \left(K^m \frac{\partial U}{\partial z} \right) \qquad (13)$$

in conjunction with (3), seemed to be preferable. The reasons for this are mathematical and computational simplicity and the fact that the mixing length models are still adequate within the surface layer.

Thus it was concluded that the problem of simultaneous heat and water vapor transfer under conditions of momentum advection, as a result of a temperature and humidity discontinuity at x = 0, may be described by equations (13), (3) and simplified forms of (5) and (6), namely

$$U \frac{\partial T}{\partial x} + W \frac{\partial T}{\partial z} = \frac{\partial}{\partial z} \left(K^h \frac{\partial T}{\partial z}\right) \tag{14}$$

$$U \frac{\partial q}{\partial x} + W \frac{\partial q}{\partial z} = \frac{\partial}{\partial z} \left(K^v \frac{\partial q}{\partial z}\right) \tag{15}$$

Note that in contrast to equations (11) and (12) the terms in W are retained because of the effect of momentum advection. The diffusivities were assumed to be given by empirical but mathematically convenient functions of the stability of the Dyer - Businger type.

Numerical solutions of this set of equations showed that the sudden instability of the flow initially can greatly increase the efficiency of the turbulent exchange processes at the surface and that the vapor flux can greatly contribute to the atmospheric stabiltiy. The solution was shown to be relatively insensitive to the exact form of the Monin-Obukhov similarity functions or the exactness of Reynolds' analogy or von Karman's constant. In the formulation of the problem it is essential to keep the eddy diffusivities bounded at the value corresponding to the top of the atmospheric surface layer. Blackadar's modification of the mixing length provides a suitable way of accomplishing this. For large fetches and near neutral conditions the solution becomes similar to that obtained in Sutton's problem and with Harbeck's empirical formula. The details of this study are given in Section X, Appendix.

SECTION VI

PREDICTION OF EVAPORATION FROM A LARGE WATER BODY

In view of the theoretical findings described above, evaporation from a free water surface can be calculated, with a degree of accuracy sufficient for most engineering problems, by means of the following equation

$$E = E_a + b\, U_z\, (e_o - e_a) \qquad (16)$$

where E is the average evaporation in cm/sec, e_o and e_a are the vapor pressure in millibars at the water surface and in the air unaffected by the water surface (i.e. upwind from the water body), respectively, U_z is the wind speed in cm/sec at an elevation z, b is the so-called mass transfer coefficient and E_a is the evapotranspiration from the land surrounding the water body. Similarly sensible heat flux in(cal/cm^2 sec) may be estimated from

$$H = H_a + (b\, p\, c_p/0.622)\, U_z\, (T_o - T_a) \qquad (16_*)$$

where c_p = 0.24 (cal/g °K) is the specific heat at constant pressure, T_w and T_a the temperature of the water surface and the air, respectively, in °C and p the pressure in mb.

From a practical point of view the main problem is the determination of the coefficient b. In general, it is a fairly complicated function of the stability of the air, the direction of the wind, the topography and the nature of the surrounding land, the size and the geometry of the evaporating surface, and numerous other factors which can only partly be included in the presently available theoretical models. Therefore, as it is very difficult to ascertain b on a purely theoretical basis, for a particular water body, lake or even stream, it is probably best determined experimentally. Harbeck and Meyers (1970) suggested that this be accomplished by means of the energy budget method, which albeit more accurate can only be used for a short intensive measurement period on account of the generally more expensive equipment and data processing involved.

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When, however, b cannot be determined experimentally, for averaging periods of a few weeks to a month Harbeck's (1962) coefficient should be quite adequate. Thus equation (16) can be applied with U_z at 2m above the surface and with (Brutsaert and Yeh, 1970)

$$b = 5.32\ 10^{-9}\ A^{-0.05} \tag{17}$$

where A is the water surface area in cm^2. This equation was shown to agree with the forced convection model under near neutral conditions at 20°C over a surface with a roughness of approximately $z_o = 0.02$ cm.

Equation (16) with (17) takes account of the effect of horizontal advection of the wind, since e_a refers to the humidity upwind from the water body and since A is included in the formulation. Sometimes, when horizontal advection is small and under near neutral conditions the local evaporation can be calculated by means of the drag coefficient on the basis of local variables, namely

$$E = C_E\ \rho\ U_z\ (q_o - q_z) \tag{18}$$

where q_o is the specific humidity at the water surface and q_z at the level z. The local heat flux may be estimated by the analogous equation

$$H = C_E\ \rho\ c_p\ U_z\ (T_o - T_z) \tag{18*}$$

For z = 10m, and $z_o = 0.02$ cm one obtains by means of the logarithmic wind profile $C_E = 1.36\ 10^{-3}$. This is in excellent agreement with the value $1.2\ 10^{-3}$ recently observed over the ocean east of Barbados (Pond, et al., 1971). For an assumed mean temperature of 20°C, an air pressure of 1013.2 mb and a density of the air $\rho = 1.20\ 10^{-3}$ g/cc equation (18) can be written in a form similar to (16), i.e. approximately

$$E = 1.0\ \ 10^{-9}\ U_{10}\ (e_o - e_{10}) \tag{19}$$

where now U_{10} and e_{10} are the wind and the local vapor pressure at 10 m. above the surface.

Note finally that the variables used in any of these transfer equations should preferably be means taken over periods of a day or less. If, as is often done, monthly means are used to calculate the monthly mean evaporation, serious errors may result. It can easily be shown that the difference between the mean of the products of two variables and the product of the means equals the covariance between the two variables. For example Jobson (1972) made an analysis of this type of error by analyzing data obtained at Lake Hefner, Oklahoma, and found that using daily means resulted in an error of 0.088% with a variance of 29%, and monthly means in an error of 5.7% with a variance of 30%.

SECTION VII

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SECTION X

APPENDIX

Heat and Water Vapor Transfer from a Water Surface under Unstable Atmospheric Conditions

by

Richard N. Weisman and Wilfried Brutsaert

Formulation of the Model. The problem under consideration is one of steady turbulent flow of air from over a homogeneous land surface but with a different temperature and specific humidity. The mean motion over the upwind land surface is plan-parallel and a function of elevation z only. Furthermore, turbulent diffusion in the direction lateral to the mean wind is known to be negligible in the calculation of evaporation from large water surfaces (Brutsaert and Yeh, 1969). Therefore, the problem may be reduced to two dimensions like flow over an infinitely long strip at right angles to the mean wind. As the air moves over this surface discontinuity in temperature and humidity, an internal boundary layer develops within which the mean wind is no longer strictly horizontal as it is upwind. The horizontal component, u, and the vertical component, w, of the mean wind may be related through the continuity equation (3).

Under conditions of fully turbulent flow, molecular diffusion is negligibly small. Since the effect of turbulent diffusion in the direction of the wind has also been found to be negligible, sensible heat and water vapor satisfy the conservation equations (14) and (15), respectively.

Evaporation and heat flux from the water surface are controlled primarily by the motion in the atmospheric surface layer. Within this layer the effects of the pressure gradient and of the earth's rotation, producing the Coriolis acceleration, are neglibible. Also, near the surface the motion is very close to horizontal, so that w is probably at least an order magnitude smaller than u. Thus it is permissible to neglect vertical momentum in accordance with Prandtl's boundary layer assumption. This means that horizontal momentum takes on the properties of a conservative scalar admixture of the flow and that the horizontal Reynolds equation reduces to (13) which is similar to (14) and (15). This also implies that no consideration

is given here to convective cells or return currents aloft associated with heat islands or meso-scale sea breeze problems. In orther words, buoyancy effects are not analyzed by a vertical equation of motion; rather they are considered only as they affect the vertical profile of the mean horizontal wind and the eddy diffusivities by means of the Obukhov stability length.

In the system of equations (3), (13), (14) and (15) use was made of the semi-empirical theory of turbulence. However, there are numerous ways to express the required eddy diffusivities in terms of the mean quantities. Monin and Obukhov's (1954) semi-empirical similarity model for the lower atmosphere suggests that they be of the form

$$K^v = u_* \, \ell/\phi_V \tag{A1}$$

$$K^h = u_* \, \ell/\phi_H \tag{A2}$$

$$K^m = u_* \, \ell/\phi_M \tag{A3}$$

where $u_* = \sqrt{\tau/\rho}$ is the friction velocity, ℓ the mixing length or a length scale characteristic of the turbulence and the ϕ's are supposedly universal functions of (z/L). The parameter L is Obukhov's (1946) (Businger and Yaglom, 1971) stability length.

To apply equations (A1), (A2), and (A3), suitable expressions must be adopted for the quantities u_*, ℓ, L and the ϕ's. Thus, in contrast to the similarity models for turbulent flow over a homogeneous surface in which the friction velocity refers to the shear stress at the surface, herein $u_* = u_*(x,z)$ refers to the local shear stress. On account of (A3), this local friction velocity can be obtained from

$$u_* = (\ell/\phi_M) \, \partial u/\partial z \tag{A4}$$

Close to the surface the mixing length is usually defined as $\ell = kz$ or also as $\ell = k(z + z_o)$ where k is von Karman's constant and z_o the roughness length. As will be shown below, the upper boundary condition at any x consists of the attainment of the upwind conditions aloft at some undetermined elevation outside the developing internal boundary layer. Therefore to prevent the increase of ℓ, and thus the eddy diffusivities, without bounds,

Blackadar's (1962) modification was adopted, viz.

$$\ell = k(z + z_o)/[1 + k(z + z_o)/\lambda] \qquad (A5)$$

where λ is some length scale, which constitutes the upper limit for ℓ. As originally proposed by Blackadar (1962), the expression for λ is

$$\lambda = .0027 \frac{V_g}{f z_o}$$

where V_g is the geostrophic or free stream wind above the boundary layer, f is the Coriolis parameter which is a function only of the earth's rate of rotation and the latitude. Taylor (1969) proposed an expression for the scaling height identical to Blackadar's except with the constant equal to .0004. If $f = 10^{-4}$/sec, z_o = .02 cm, and V_g = 500 cm/sec, the dimensionless scaling height has a value of around 7×10^4. Values of 5×10^4 and 7×10^4 were used for most calculations and they worked well as will be discussed further.

The evaluation of the effect of the density stratification due to water vapor was one of the main objectives of the study. The Obukhov stability length concept was extended to also include water vapor concentration gradient as worked out by Zilitinkevich (1966), namely

$$L = \frac{-u_*^3 \rho}{kg[(H/T c_p) + 0.61 E]}$$

where T is some reference temperature, taken as the upwind surface temperature.

Although the exact form of the Monin-Obukhov similarity functions is still open to much conjecture (e.g. Monin and Yaglom, 1971), it was decided to adopt a recent formulation which has a good experimental basis in the absence of advection and which has the advantage of mathematical simplicity. The functions were suggested by Businger (1966), Dyer (1967) and then Dyer and Hicks (1970) as follows:

$$\phi_M = (1 - B\frac{z}{L})^{-1/4} \qquad \text{(A7)}$$

$$\phi_V = \phi_H = (1 - B\frac{z}{L})^{-1/2} \qquad \text{(A8)}$$

where B, a constant, has been measured to be approximately 16. It should be noted that as the form of these ϕ-functions is the subject of continued controversy, it is not the purpose of this study to give credibility to one formulation rather than to another. Nevertheless, it is one of the objectives to test the sensitivity of the results to various values of the parameter B and also of the power in (A7). Therefore the following alternative formulation is also considered.

$$\phi_M = (1 - B\frac{z}{L})^{-1/2} \qquad \text{(A9)}$$

in which the power is in closer agreement with -0.46 obtained experimentally by Morgan, et al. (1971), and which is in accordance with Reynold's analogy $\phi_M = \phi_H = \phi_V$.

Equations (3), (13), (14) and (15) are subject to boundary conditions that may be specified from the following considerations:

(i) the incoming air is neutral and it has a humidity profile, $q_a(z)$, which is known and which results from equilibrium conditions dicated by the heat flux and the evapotranspiration from the windward land surface. The neutral velocity profile upwind from the water body is given by

$$u = u_a(z) = \frac{u_{*a}}{k} \left[\ln \left(\frac{z + z_o}{z_o}\right) + \frac{kz}{\lambda}\right] \qquad \text{(A10)}$$

where the subscript a refers to upwind conditions. Equation (A10) results from equation (A3) for a constant friction velocity and for Blackadar's (1962) mixing length of equation (A5).

(ii) High above the water surface, that is at an elevation well above the height z_h of the internal boundary layer the conditions are the same as upwind and unaffected by the water body.

(iii) at the water surface the temperature is known and uniform; the specific humidity is saturated and a unique function of this temperature.

These boundary conditions can be written as

$$x = 0;\ z > 0; \qquad q = q_a(z)$$
$$T = T_a(z)$$
$$u = u_a(z) \qquad \text{(A11)}$$

$$x > 0;\ z > z_h; \qquad q = q_a(z)$$
$$T = T_a(z)$$
$$u = u_a(z) \qquad \text{(A12)}$$

$$x > 0;\ z = 0; \qquad q = q_w^o$$
$$T = T_w^o$$
$$u = w = 0 \qquad \text{(A12)}$$

where the subscript w refers to conditions over the water and the superscript o to surface conditions at z = 0.

Solution of the System. In order to give the equations and their solution a more universal meaning, they are made dimensionless with the following non-dimensional quantities:

$$\hat{u} = u/u_{*a},\ \hat{w} = w/u_{*a},\ \hat{u}_* = u_*/u_{*a},\ \hat{\tau} = \hat{u}_*^2$$

$$\hat{x} = x/z_o,\ \hat{z} = z/z_o,\ \hat{\lambda} = \lambda/z_o,\ \hat{L} = L/z_o,\ \hat{\ell} = \ell/z_o$$

$$\hat{\theta} = \frac{T - T_a}{T_w - T_a^o},\ \hat{Q} = \frac{q - q_a}{q_w - q_a^o}$$

$$\hat{H} = \frac{H}{\rho c_p u_{*a} (T_w - T_a^o)},\quad \hat{E} = \frac{E}{\rho u_{*a} (q_w - q_a^o)}$$

Also, the vertical axis is transforemed logarthmically to $\hat{\zeta} = \ln(\hat{z} + 1)$ as suggested by Taylor and Delage (1971).

Equations (3), (13), (14), (15) and (A5) to (A8) become

$$\frac{\partial \hat{u}}{\partial \hat{x}} + e^{-\hat{\zeta}} \frac{\partial \hat{w}}{\partial \hat{\zeta}} = 0 \tag{A14}$$

$$\hat{u} \frac{\partial \hat{u}}{\partial \hat{x}} + \hat{w} e^{-\hat{\zeta}} \frac{\partial \hat{u}}{\partial \hat{\zeta}} = e^{-\hat{\zeta}} \frac{\partial \hat{\tau}}{\partial \hat{\zeta}} \tag{A15}$$

$$\hat{u} \frac{\partial \hat{Q}}{\partial \hat{x}} + \hat{w} e^{-\hat{\zeta}} \frac{\partial \hat{Q}}{\partial \hat{\zeta}} = -e^{-\hat{\zeta}} \frac{\partial \hat{E}}{\partial \hat{\zeta}} \tag{A16}$$

$$\hat{u} \frac{\partial \hat{\theta}}{\partial \hat{x}} + \hat{w} e^{-\hat{\zeta}} \frac{\partial \hat{\theta}}{\partial \hat{\zeta}} = -e^{-\hat{\zeta}} \frac{\partial \hat{H}}{\partial \hat{\zeta}} \tag{A17}$$

$$\hat{u}_* = \frac{\hat{\ell} e^{-\hat{\zeta}}}{\phi_M} \frac{\partial \hat{u}}{\partial \hat{\zeta}} \tag{A18}$$

$$\hat{E} = \frac{\hat{\ell} \hat{u}_* e^{-\hat{\zeta}}}{\phi_H} \frac{\partial \hat{Q}}{\partial \hat{\zeta}} \tag{A19}$$

$$\hat{H} = \frac{\hat{\ell} \hat{u}_* e^{-\hat{\zeta}}}{\phi_V} \frac{\partial \hat{\theta}}{\partial \hat{\zeta}} \tag{A20}$$

$$\hat{\ell} = \frac{k e^{\hat{\zeta}}}{1 + k e^{\hat{\zeta}}/\lambda} \tag{A21}$$

$$\phi_M = (1 - B \frac{\hat{z}}{\hat{L}})^{-1/4} \tag{A22}$$

$$\phi_V = \phi_H = (1 - B \frac{\hat{z}}{\hat{L}})^{-1/2} \tag{A23}$$

$$\hat{L} = \frac{\hat{u}_*^3}{A_* \hat{H} + B_* \hat{E}} \tag{A24}$$

The boundary conditions, equations (A11), (A12), and (A13) are now

$$\hat{x} = 0;\ \hat{\zeta} > 0; \qquad \hat{Q}(\hat{\zeta}) = 0$$

$$\hat{\theta}(\hat{\zeta}) = 0$$

$$\hat{u}(\hat{\zeta}) = \frac{\hat{\zeta}}{k} + \frac{z}{\hat{\lambda}} \tag{A25}$$

$$\hat{x} > 0;\ \hat{\zeta} > \hat{\zeta}_h; \qquad \hat{Q}(\hat{\zeta}) = 0$$

$$\hat{\theta}(\hat{\zeta}) = 0$$

$$\hat{u}(\hat{\zeta}) = \frac{\hat{\zeta}}{k} + \frac{z}{\hat{\lambda}} \tag{A26}$$

$$\hat{x} > o;\ \hat{\zeta} = 0; \qquad \hat{Q} = 1$$

$$\hat{\theta} = 1$$

$$\hat{u} = \hat{w} = 0 \tag{A27}$$

The hat, ^, denoting non-dimensional quantities is now dropped for convenience.

Two dimensionless parameters appear in the stability length, L, as a result of the non-dimensionalization process, namely

$$A_* = -\left(\frac{T_w - T_a^o}{T_a^o}\right) \frac{kgz_o}{u_{*a}^2} \tag{A28}$$

$$B_* = -.61\,(q_w - q_a^o)\,\frac{kgz_o}{u_{*a}^2} \tag{A29}$$

These two parameters must be specified to solve the system of equations. Both parameters contain a term resembling an inverse Froude number, kgz_o/u_{*a}^2 providing some indication of the relative importance of gravity and inertia effects, in this case, free and forced convection. A_* contains a fractional change in surface temperature while B_* contains the change in surface specific humidity at the leading edge. A_* and B_* are referred to as the temperature change stability parameter and the specific humidity change stability parameter, respectively. Both represent a measure of the discontinuity at the leading edge.

As an illustration, consider a surface with a roughness length of z_o = .02 cm. If the wind speed at 2 m is 100 cm/sec, the logarithmic velocity profile yields a friction velocity of 5 cm/sec. An average temperature change at a leading edge might be taken as .05 and, if the upwind condition is dry, the surface specific humidity jump might be .03 kg/kg. Thus, putting the above values into equations (A28) and (A29) yields the average values: A_* = -.015 and B_* = -.005.

In the numerical solution, A_* was given values equal to 0, -0.001, -0.01, -0.05, and -0.1 and B_* was given values equal to 0, -0.001, -0.003, and -0.01. The higher values of A_* and B_* correspond to larger surface temperature and specific humidity changes at the leading edge, to a lower upwind velocity, or to a higher roughness value. These ranges of values for the stability parameters are probably larger than would be met in nature and still be applicable to the problem considered here.

The equations (A14) to (A24) with boundary conditions (A25), (A26), and (A27) were solved using an explicit Euler scheme. More sophisticated and accurate methods would incur an increase in computer time and probably are not warranted for the complex problem considered here. For a complete description of the numerical procedure, the reader is referred to Weisman (1973).

Central differences were used for vertical gradients in all the equations except in the continuity equation (A14) which was formulated using backward differences as suggested first by Peterson (1969). A difficulty arises in using central differences because it thereby becomes impossible to define the gradients and, therefore, the fluxes at the surface.

By making certain gross assumptions, equations (A15), (A16), and (A17) can be simplified to heat-type equations for which an approximate stability analysis can be made. The maximum allowable downwind step size derived from the stability criteria is a function of the velocity at the first vertical grid point and the distance to that grid point from the surface. For reasonable vertical grid spacings, the required horizontal step size is quite small which necessitates an unreasonable amount of computer time to move a physically relevant distance downwind. This and the aforementioned difficulty were resolved with the so-called constant flux wall layer.

In this scheme it is assumed that a thin layer of air exists next to the surface in which the vapor vlux, E_w, heat flux, H_w, and friction velocity, u_w, are constant with height. This constant flux layer was used by Peterson (1969) and Taylor (1970); Taylor and Delage (1971) have discussed this method.

The first difficulty, namely that of defining the fluxes at the surface, is resolved since the flux equations (A18), (A19), and (A20) can be integrated analytically to obtain the profiles of humidity, temperature, and velocity within the constant flux layer; the fluxes may then be calculated by knowing the values of humidity, temperature, and velocity at the top of the layer. These values were obtained from the Euler method.

If now the height of the constant flux layer is allowed to grow as the internal boundary layer grows, the second difficulty, that of small forward step sizes, is also resolved. Thus, the forward step sizes can be increased considerably allowing a gradually faster numerical solution as it moves downwind. Note also that though there is little physical justification for a constant layer under advecting conditions, it can be shown from equations (A18), (A19) and (A20) that the fluxes become constant as z approaches zero. Moreover, as the air mass moves downwind, equilbrium conditions become established in which horizontal gradients tend to zero and the vertical fluxes are constant with height.

Results. Averaging the local vapor flux at the surface, $\overline{E_w(x)}$, over the fetch of the water, x_o, one obtains for the total average evaporation

$$W = \frac{1}{x_o} \int_o^{x_o} E_w dx$$

Since the extreme case is the most interesting one, the results presented are for upstream conditions neutral and dry. For these conditions, the total average heat flux is defined by

$$HF = \frac{1}{x_o} \int_o^{x_o} H_w dx$$

Similarly, dimensionless temperature, θ, and specific humidity, Q, are numerically equal for neutral, dry upwind conditions.

Figure 1 is a curve of total average vapor flux, W, versus the fetch of the water surface, x_o, for various values of the A_* stability parameter while B_* is held at a constant average value. Figure 2 is the same as figure 1 except that A_* is kept constant at an average value and B_* is the parameter. Figures 3 and 4 plot the above results for A_* equal to zero and B_* varying, and B_* equal to zero with A_* varying.

As can be seen from the figures 1 through 4, the total average evaporative flux decreases as the fetch increases. Also, evaporation is greater for the larger values of A_* and B_*; that is, when the instability created by the temperature and/or specific humidity change at the leading edge is large, the evaporation is large. The effect of A_*, the surface temperature stability parameter, is quite significant. The effect of the surface specific humidity change parameter, B_*, is smaller than that of A_*, but there is still a twenty percent increase in evaporation at $x_o = 10^7$ from $B_* = -.001$ to $B_* = -.01$ as seen in figure 2.

From the effect of B_*, the specific humidity stability parameter, on the evaporation, as seen in figures 2 and 4, it can be stated that the concentration buoyancy term in the modified Obukhov length, equation (A6), has a significant impact on the momentum exchange. Hence, even when no temperature discontinuity occurs between the land and water surfaces, the air flow does not remain neutral; rather the flow becomes unstable and it does affect the evaporative flux significantly.

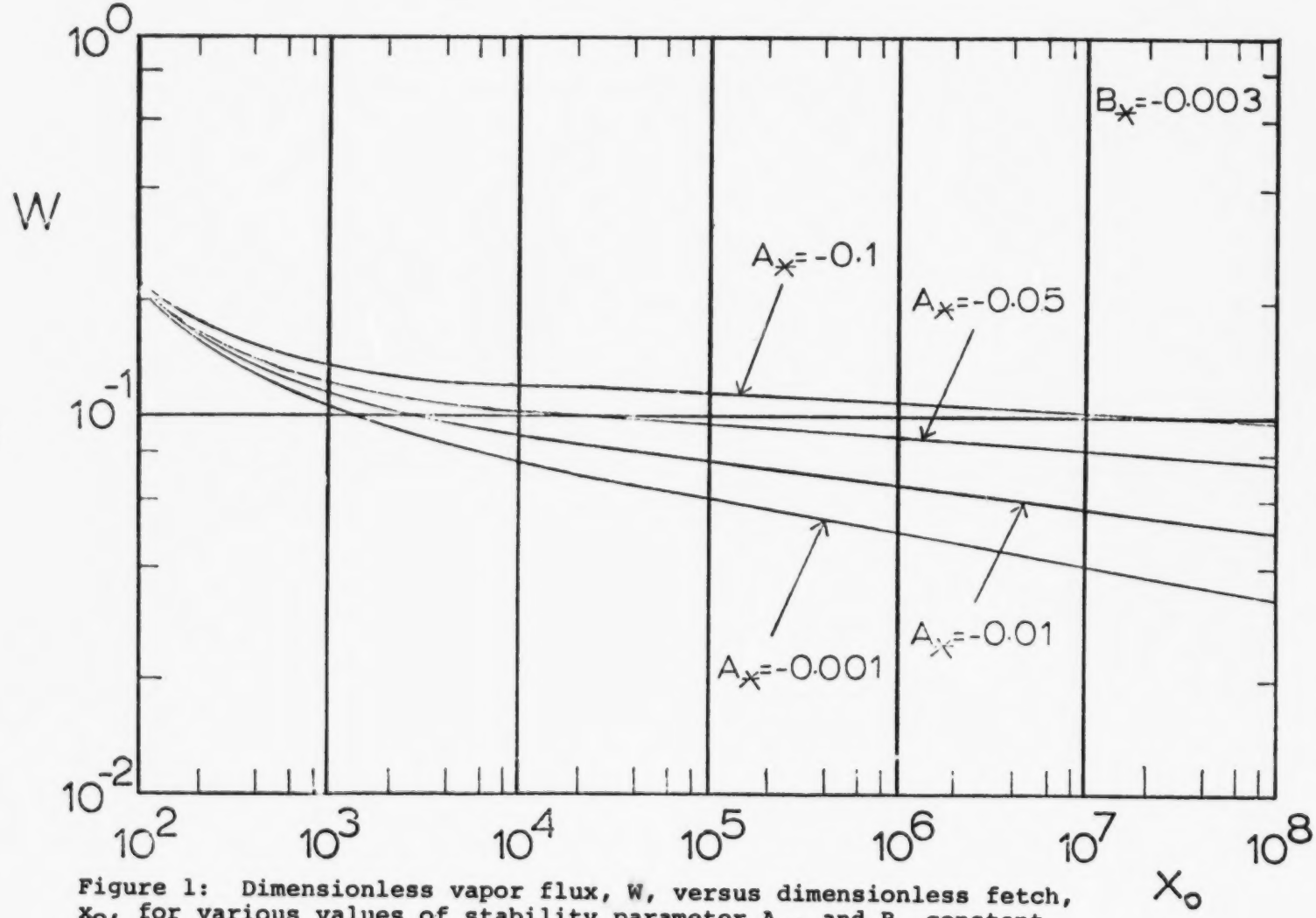

Figure 1: Dimensionless vapor flux, W, versus dimensionless fetch, x_o, for various values of stability parameter A_*, and B_* constant.

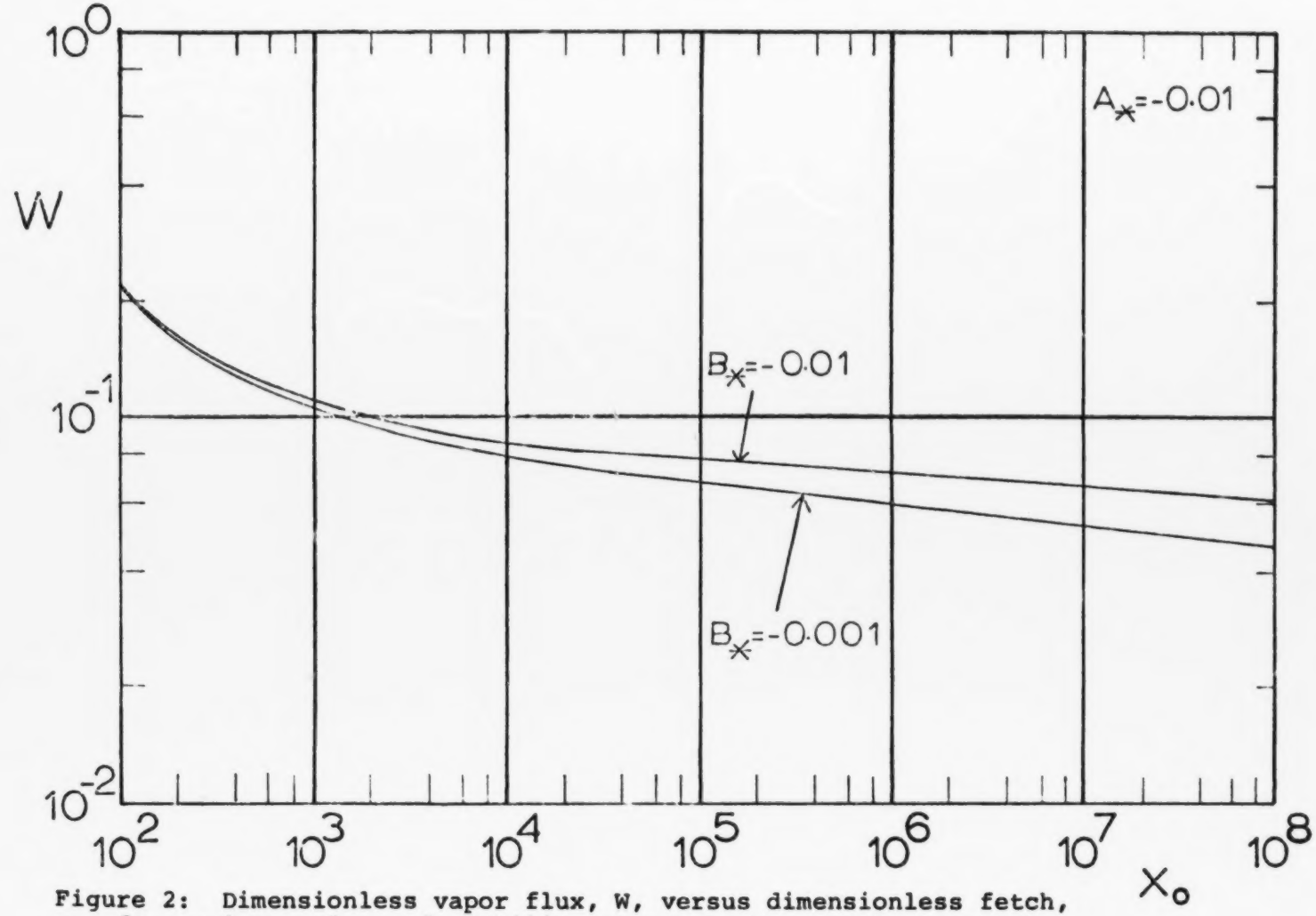

Figure 2: Dimensionless vapor flux, W, versus dimensionless fetch, x_o, for various values of stability parameter B_*, and A_* constant.

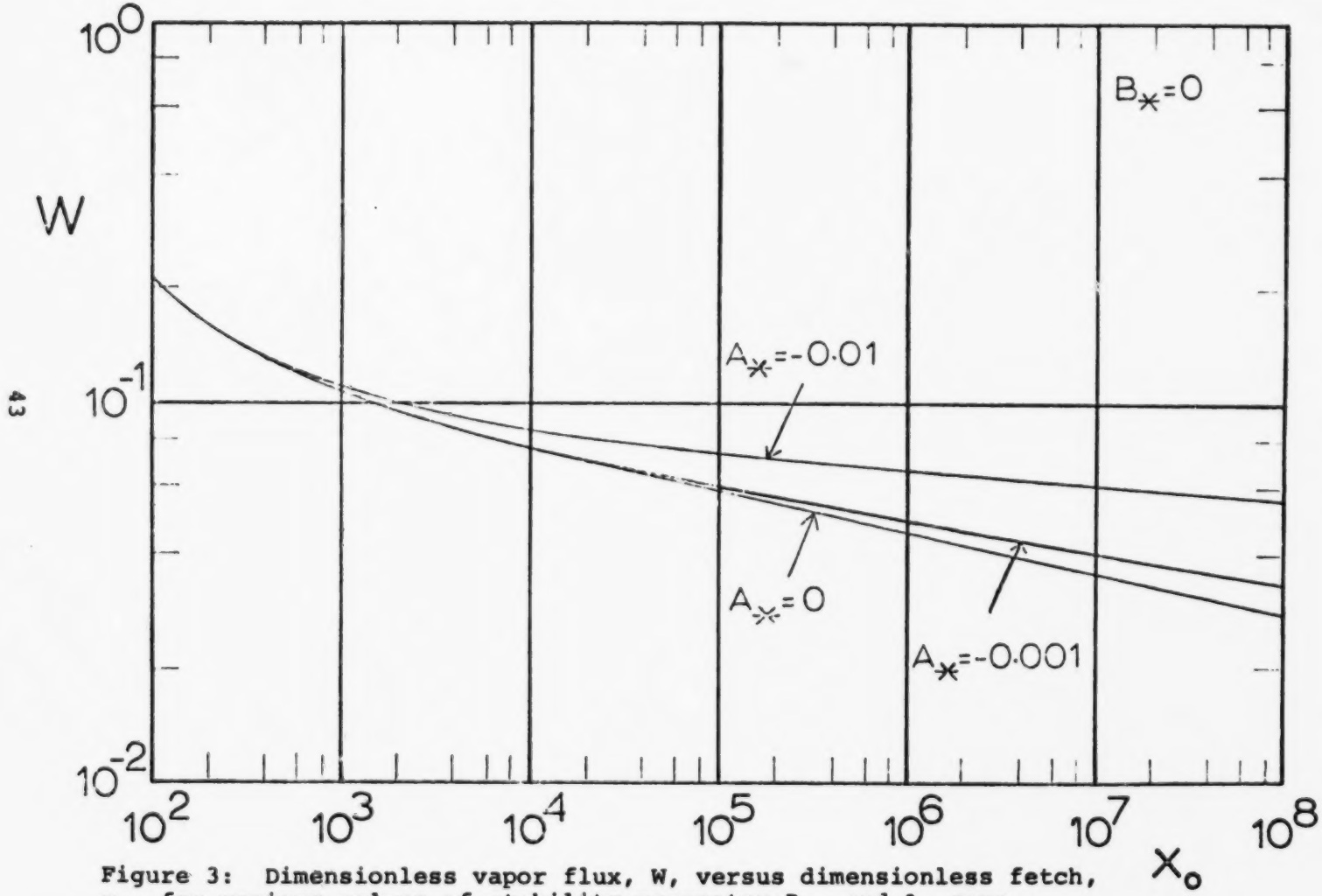

Figure 3: Dimensionless vapor flux, W, versus dimensionless fetch, x_o, for various values of stability parameter B_*, and A_* zero.

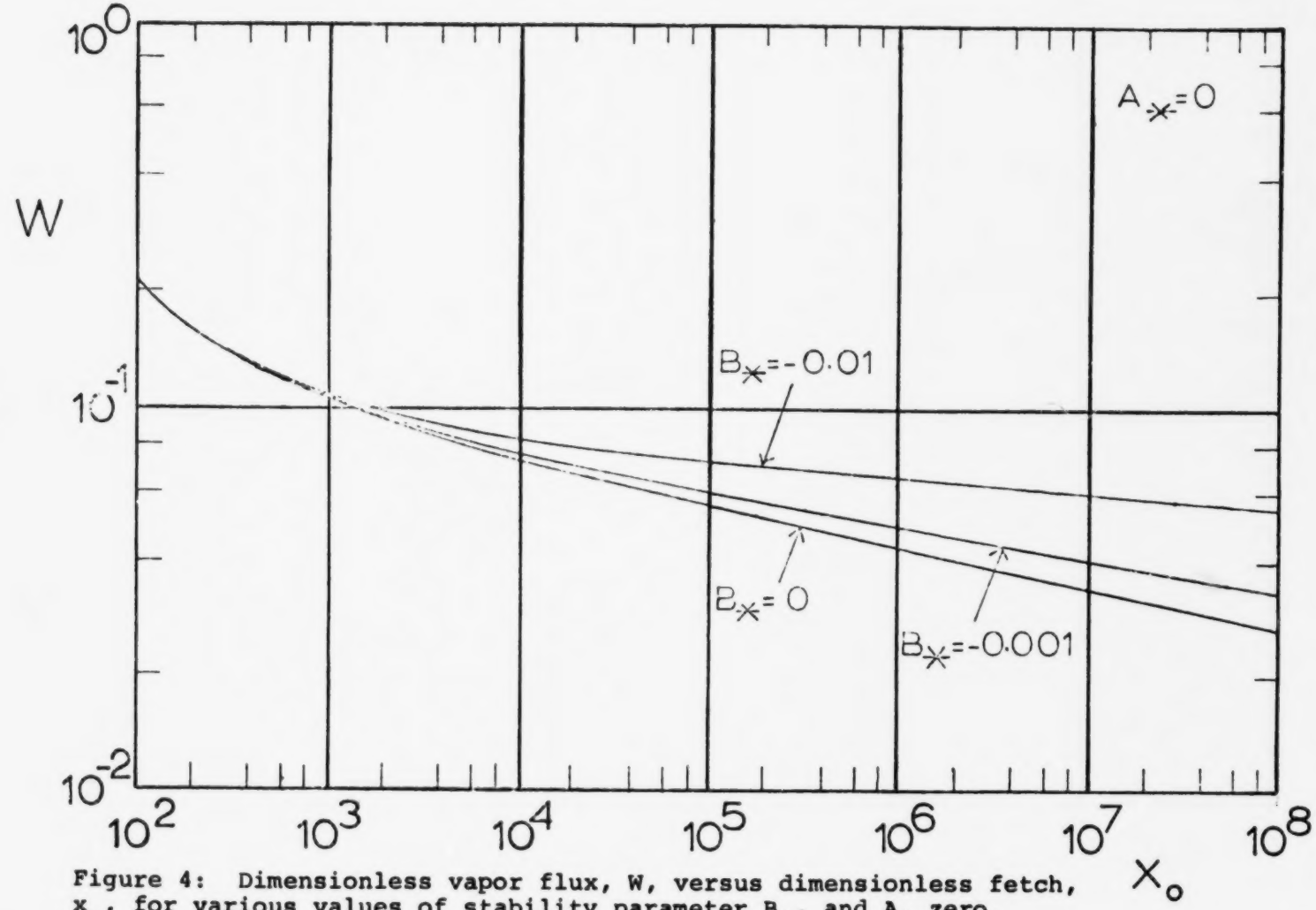

Figure 4: Dimensionless vapor flux, W, versus dimensionless fetch, x_o, for various values of stability parameter B_*, and A_* zero.

Both figures 3 and 4 show that, by allowing momentum to be advected, the solution is greatly affected. When A_* and B_* are both zero, the modified Obukhov length, L, goes to infinity and all the ϕ's become unity. Momentum is no longer advected and the velocity profile remains unchanged downwind. Note that the case $A_* = B_* = 0$ is the Sutton-type problem. The main difference with the Sutton solution is that the present case is not solved for the power law of the wind profile but for Blackadar's log-linear profile equation (A10). The related solution with the logarithmic profile has been treated by Yeh and Brutsaert (1971a).

Figure 5 is a plot of total average evaporative flux, W, versus fetch, x_o, for smaller evaporating surfaces. One curve represents the calculation performed with $\phi_M = (1 - B\ z/L)^{-1/2}$, equation (A9), instead of $\phi_M = (1 - B\ z/L)^{-1/4}$, equation (A7). As can be seen in figure 5, the solutions tend to merge at values of the fetch, x_o, greater than about 10^3; for larger values of x_o, no difference in the curves can be seen between solutions using the -1/2 exponent as compared with the -1/4 exponent in the ϕ_M function. Although only a few runs were made using the -1/2 exponent and compared with the solution using the -1/4 exponent in ϕ_M, it is probably safe to infer that no difference would occur in the solutions for any A_*, B_* combination above $x_o = 10^3$.

A reason for the above result is that, beyond the initial high instabilities at the leading edge, the stability condition downwind approaches neutral quickly and the velocity profile changes only slowly. This can be seen by expanding both equations (A7) and (A9) in a Taylor series. Thus, for small z/L, the two expressions for ϕ_M are about equal. If z/L is large (at the leading edge), the two expressions for ϕ_M are different and the results are significantly affected by the form of the dimensionless shear stress.

Figure 6 is a graph of surface shear stress, τ_w, versus downwind distance, x. As can be seen, the shear stress increases downwind, greatly for the more unstable condition of $A_* = -.1$ and less so for smaller A_*. The shear stress should not increase indefinitely. Taylor (1970a), using Prandtl's mixing length, $\ell = k\ (z + z_o)$, and giving no consideration to the problem of vapor and its effect on stability, solved a similar problem to the one considered here and found that the shear stress did increase indefinitely. By using Blackadar's mixing length, equation (A5),

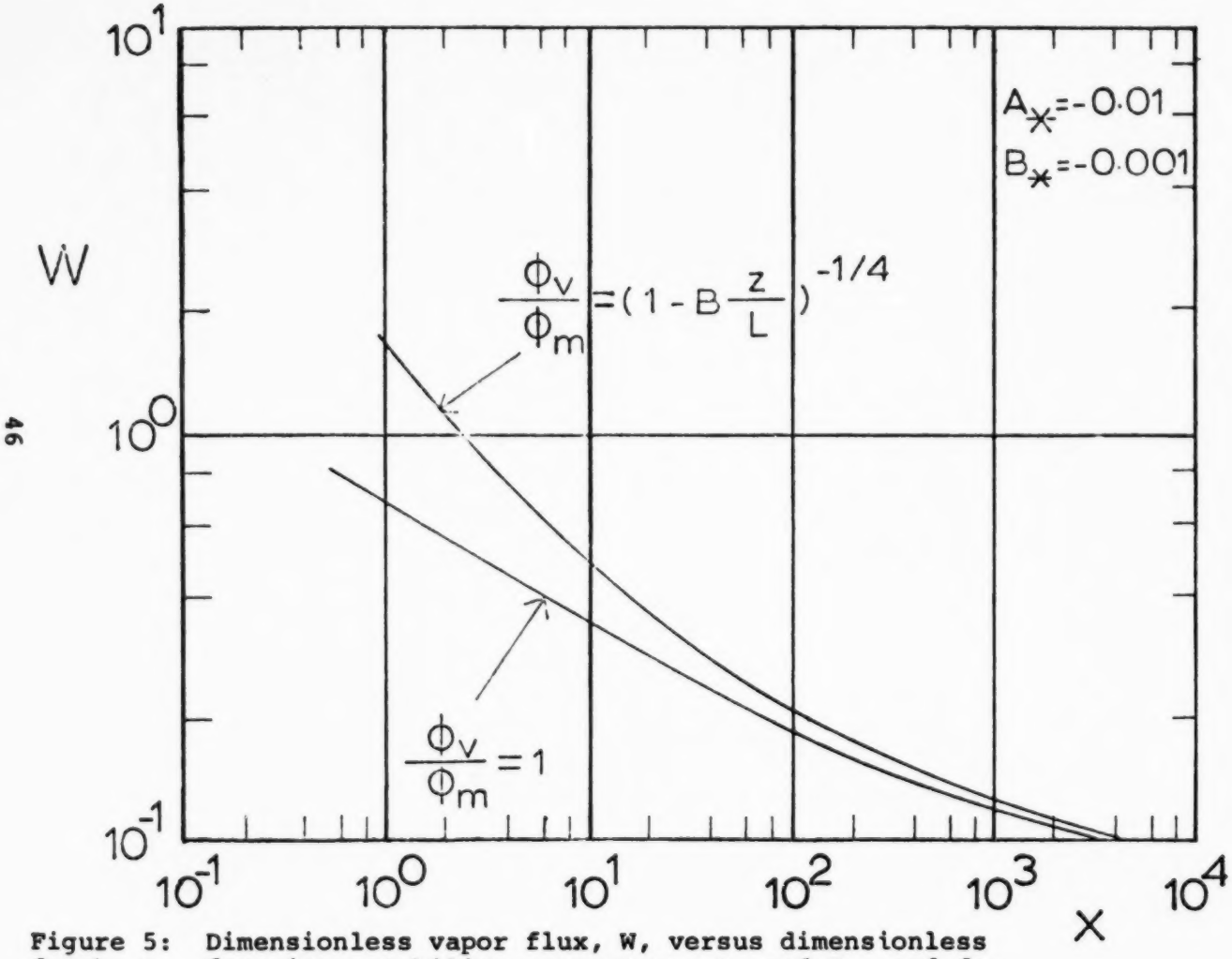

Figure 5: Dimensionless vapor flux, W, versus dimensionless fetch, x_o, for given stability parameters, A_* and B_*, and for two ratios of ϕ_V/ϕ_M.

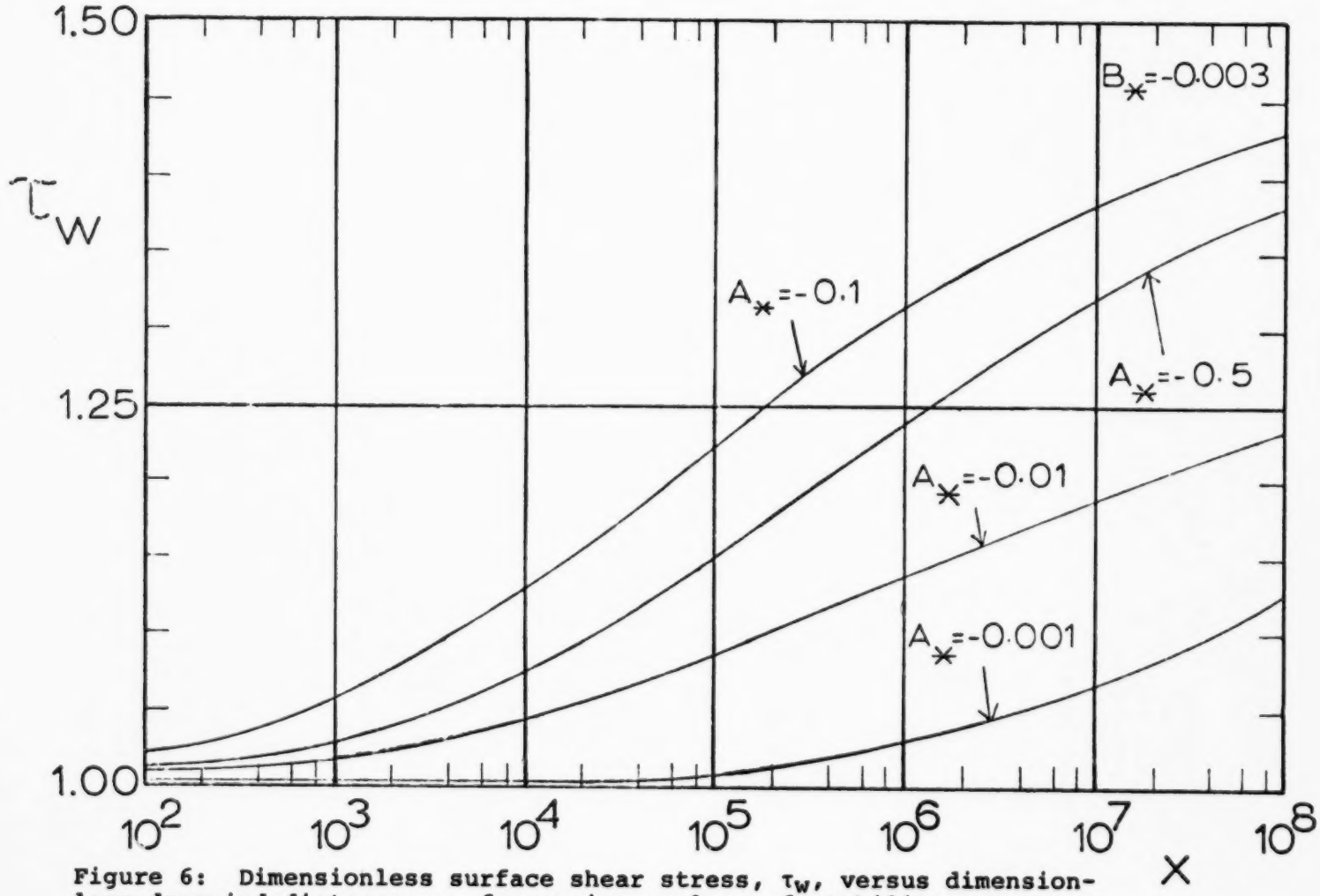

Figure 6: Dimensionless surface shear stress, τ_w, versus dimensionless downwind distance, x, for various values of stability parameter A_*, and B_* constant.

it appears from figure 6 that this difficulty is partially resolved since the shear stress curves have an inflection point and might reach an asympotic value far downwind. Nevertheless, also in the present solution the shear stress, τ_w, does not seem to tend to unity for large x whereas the wind again assumes its upwind neutral profile.

In figure 7, the negative inverse of the surface Obukhov length, that is an Obukhov length with the surface fluxes, is plotted against downwind distance for several stability conditions. This curve would indicate that the air mass is returning to neutrality as it moves downwind.

Figures 8, 9, and 10 show the effects of changing various physical and numerical constants on figure 1, i.e. total average vapor flux, W, versus the fetch, x_o. Recently, Businger et al. (1971) have concluded from experimental wind and temperature profiles under a broad range of stability conditions that von Karman's k has a value of 0.35. As can be seen in figure 8 the values of vapor flux, W, using $k = 0.35$ are only slightly lower than the results using $k = 0.4$. On the same curve, points are shown representing calculations made with various initial vertical step sizes, h, in the numerical solution. No change in the solution is visible.

Figure 9 is a curve of total average vapor flux, W, versus the fetch, x_o, for specific average stability parameters in order to test the sensitivity of the solution to the value of B, the Businger-Dyer constant in equations (A7), (A8), and (A9). The solution is quite insensitive to the value of B; as can be seen from figure 9, a jump from $B = 12$ to $B = 20$ results in a twenty percent increase in evaporation for a wet surface with a fetch of $x_o = 10^7$. However, it is doubtful that B ever has a value as low as 12 or as high as 20. In the literature, values of 15 or 16 have been reported and used. The conclusion can be drawn that any realistic value of the Businger-Dyer constant or the constant in the O'KEYPS equation (Businger and Yaglom, 1971) has little effect on the calculation of evaporation.

In figure 10, total average evaporative flux, W, is plotted against fetch, x_o, with average stability parameters for various values of the scaling height, λ. Within a band of

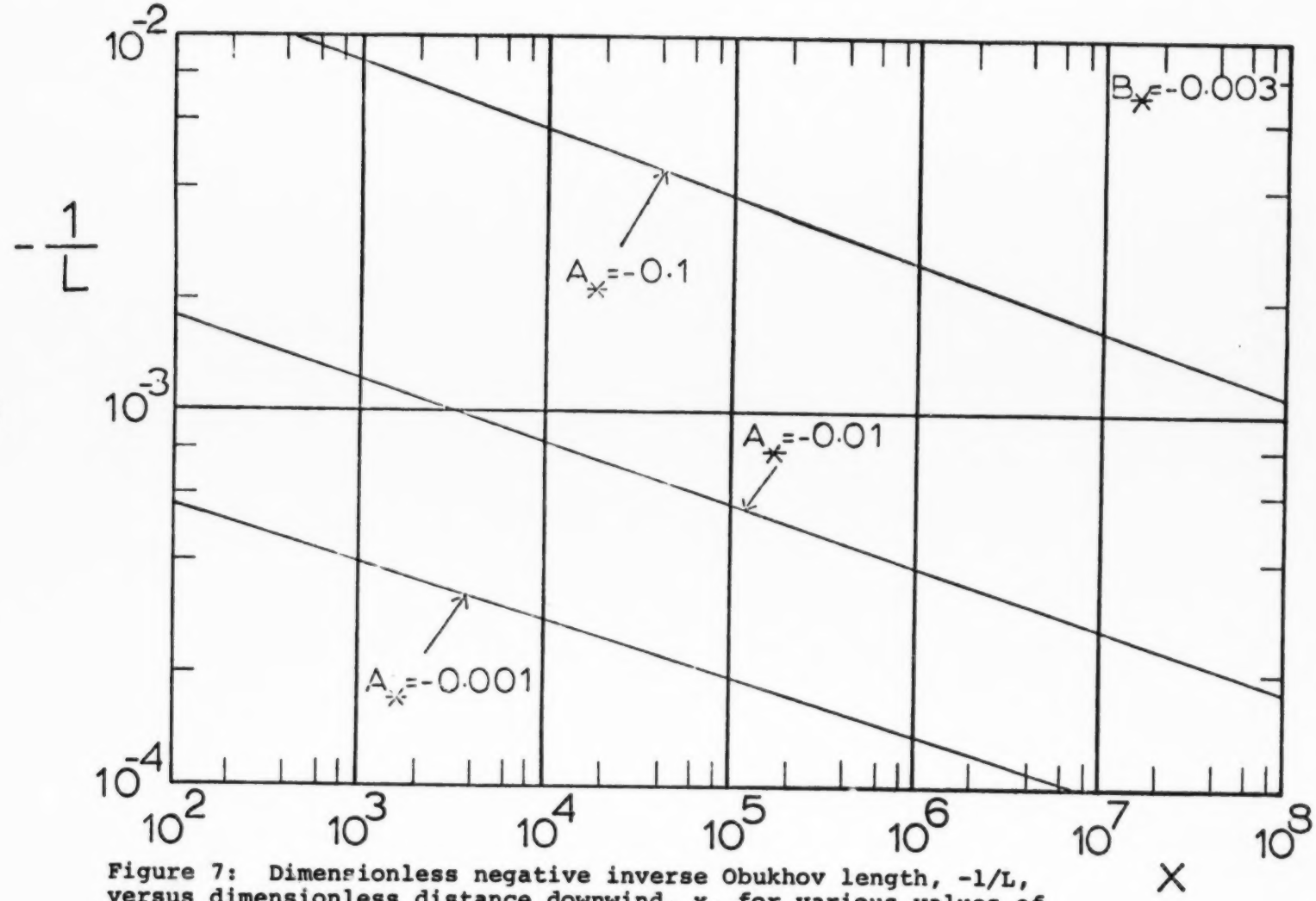

Figure 7: Dimensionless negative inverse Obukhov length, -1/L, versus dimensionless distance downwind, x, for various values of A_*, and B_* constant.

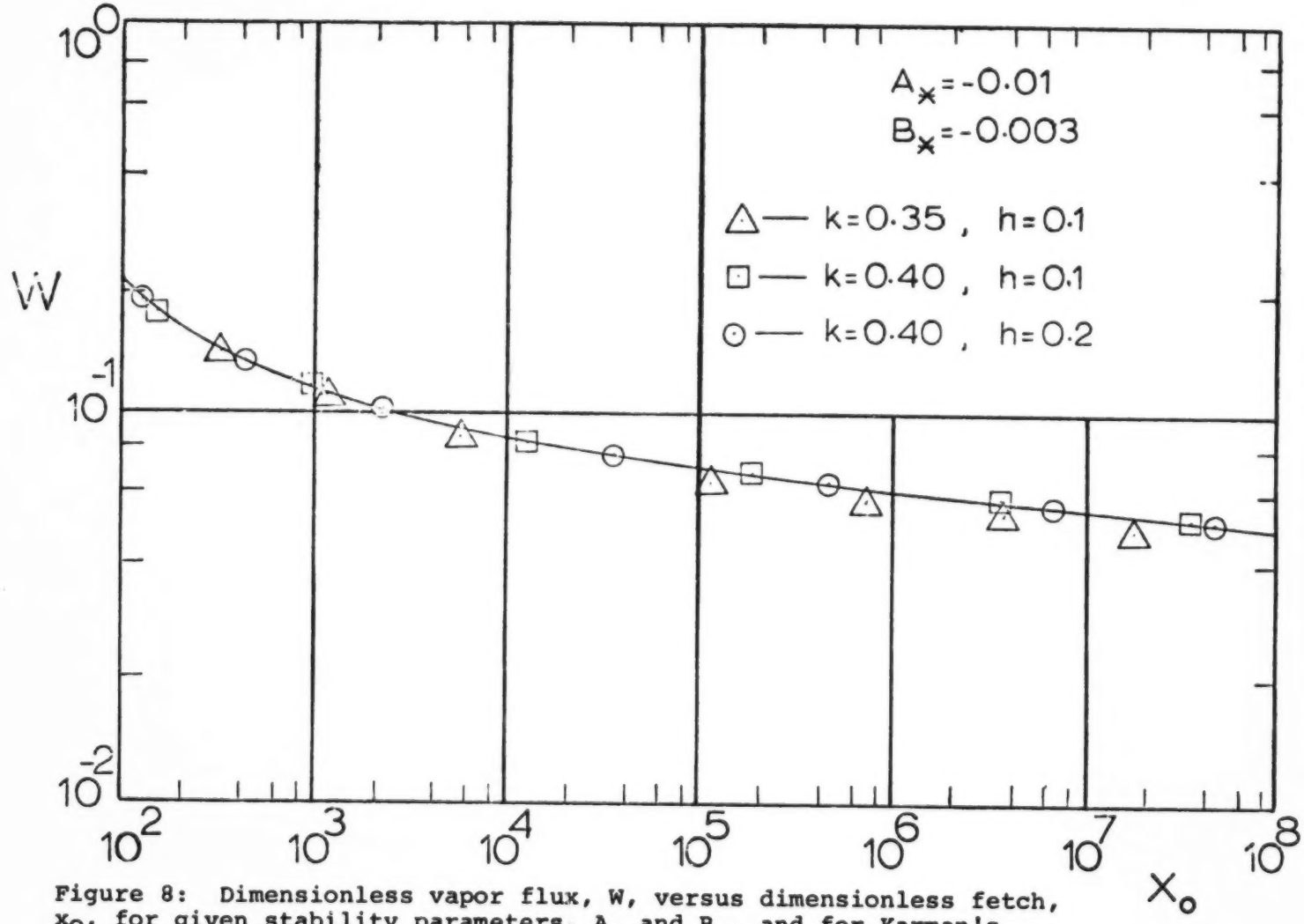

Figure 8: Dimensionless vapor flux, W, versus dimensionless fetch, x_o, for given stability parameters, A_* and B_*, and for Karman's k = .35, and two values of initial vertical step size, h.

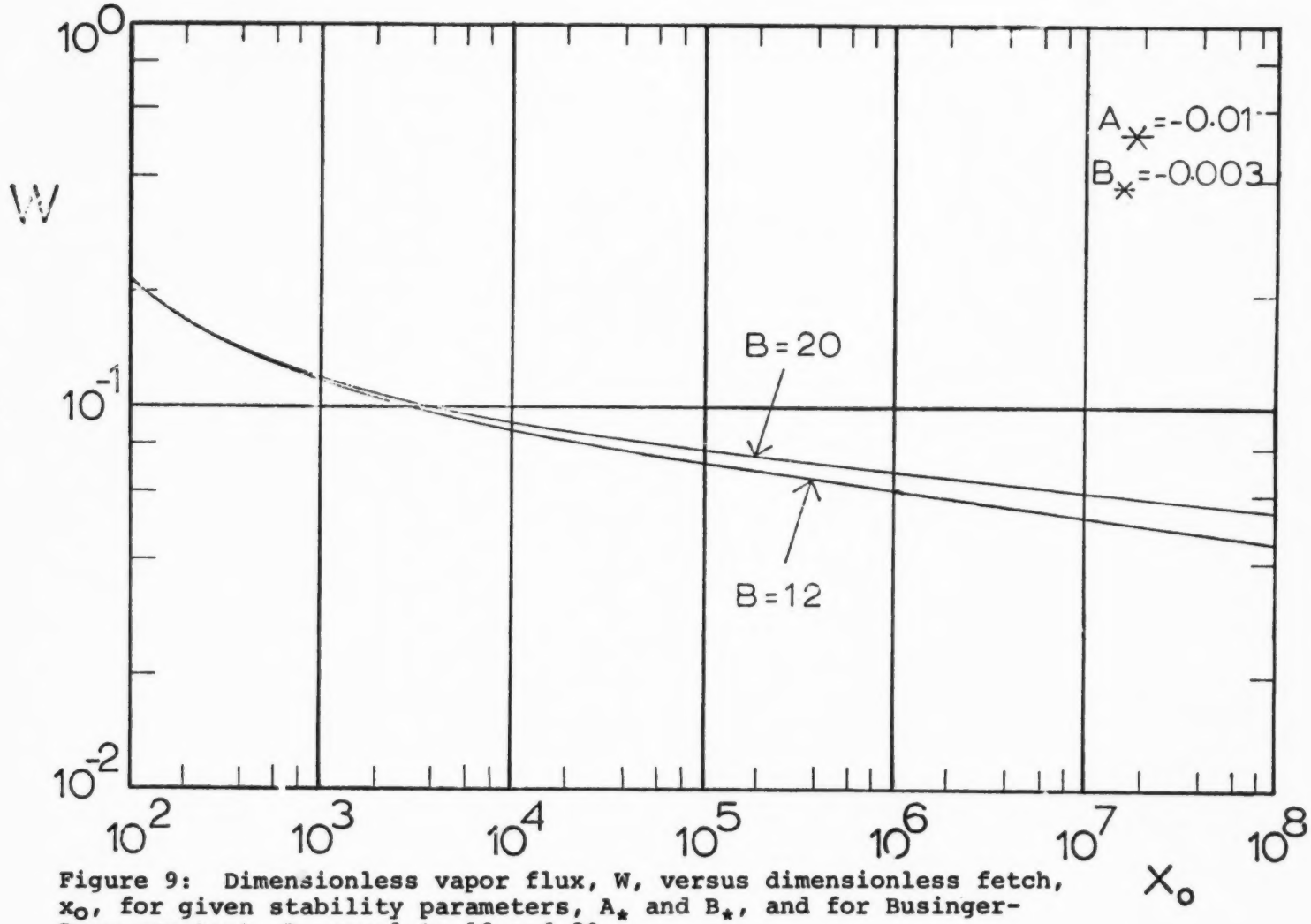

Figure 9: Dimensionless vapor flux, W, versus dimensionless fetch, x_o, for given stability parameters, A_* and B_*, and for Businger-Dyer constant, B, equal to 12 and 20.

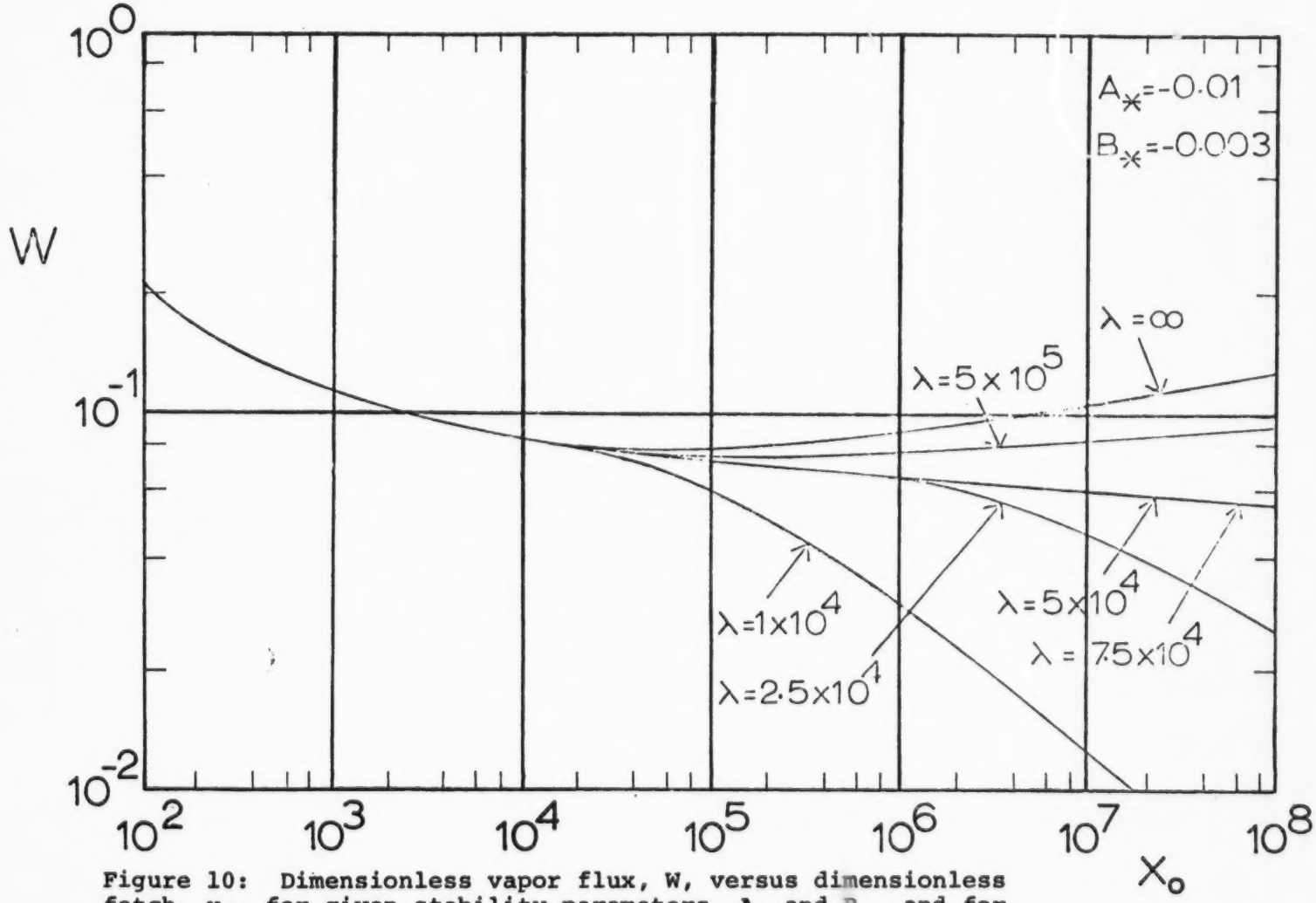

Figure 10: Dimensionless vapor flux, W, versus dimensionless fetch, x_o, for given stability parameters, A_* and B_*, and for various values of scaling height, λ, in the Blackadar mixing length.

λ-values, 5×10^4 to 1×10^5, the solution tends to stay along the same curve for large values of x_o. As mentioned previously, Taylor (1970 a,b) used Prandtl's mixing length ($\lambda \to \infty$) for the temperature jump problem only, and his solution began to increase around $x = 10^5$. Similarly, as shown in figure 10 the use of finite λ-values greater than 1×10^5 causes the solution to rise but this rise takes place further downwind and less steeply. For values of λ below 5×10^4, the solution remains on the general curve but then drops off quite suddenly. Again, the smaller the value of λ, the sooner and the more steeply the solution begins to drop downwind. It is not immediately obvious why only a range of scaling height values allows a solution further downwind. However, the values of the scaling height are well within the range reported in the literature for reasonable values of the pertinent physical quantities involved in the calculation of λ.

Discussion. For evaporating fetches, x_o, greater than about 10^4 in figures 1, 2, 3, and 4, the total average vapor flux, W, seems to become a straight line function of x_o on log-log paper. Therefore, W can be expressed as a simple power relationship

$$W = a\, x_o^{-n} \tag{A30}$$

where a and n are dependent only on a set of stability parameters A_*, B_*. The relationship of equation (A30) was determined graphically from the curves in figures 1, 2, 3, and 4 and for several other A_*, B_* combinations. The coefficient, a, is given in Table 1 and the exponent, n, is given in Table 2 for the various A_*, B_* combinations.

The solutions of this study as expressed by equation (A30) can indirectly be compared to the simpler problem considered by Sutton (1934). The Sutton solution for a non-changing velocity profile expressed as a power law, $u = c\, z^m$, is

$$W = b\, x_o^{-r} \tag{A31}$$

where

$$b = \frac{1}{\Gamma(r)\; r^{1-2r}\; (1-r)^m r\, c^{1-2r}}$$

Table 1: Values of the coeffficient a in $W = a\, x_o^{-n}$

$-B_*$ \ $-A_*$	.1	.05	.01	.001	0
.01			.121	.112	.122
.003	.150	.140	.120	.135	.166
.001			.112	.152	.167
0			.120	.167	.210

Table 2: Values of the exponent n in $W = a\, x_o^{-n}$

$-B_*$ \ $-A_*$	.1	.05	.01	.001	0
.01			.036	.042	.045
.003	.025	.034	.046	.081	.089
.001			.042	.082	.093
0			.045	.093	.112

Table 3: Values of $m = n/(1-2n)$

$-B_*$ \ $-A_*$	.1	.05	.01	.001	0
.01			.037	.046	.048
.003	.027	.037	.051	.096	.109
.001			.046	.098	.114
0			.048	.114	.144

and

$$r = \frac{m}{1 + 2m} \tag{A32}$$

The Γ denotes the gamma function, and c and m are the coefficient and exponent in the power law.

Under neutral atmospheric conditions, it has been found that m in the velocity profile power law equals approximately 1/7 or 0.143. For superadiabatic conditions, m decreases. If n in equation (A30) is set equal to r in equation (A31), equation (A32) can be solved for m. Values of m are tabulated in Table 3. For $A_* = B_* = 0.0$, m has a value of 0.144 which corresponds almost exactly to the neutral value of 1/7. Thus the case $A_* = B_* = 0$ is in agreement with the Sutton solution. Also, Table 3 shows values of m decreasing for increasingly negative values of A_* and B_*, i.e. increasingly unstable conditions. This is not unexpected since the parameter m does indeed decrease for more unstable velocity profiles (e.g. Brutsaert and Yeh, 1970b).

An empirical formula for lake evaporation, obtained by Harbeck (1962) from field measurements, can also be expressed in the form of equation (A30). The Harbeck formula is

$$E = 5.32 \times 10^{-9} \, (e_w - e_a) \, u \, (A^{1/2})^{-.1} \tag{A33}$$

where A is the area of the lake in cm^2, u the wind speed in cm/sec at two meters, e_w and e_a the vapor pressures at the water surface and in the upwind air, respectively, in millibars, and E the mean evaporative flux in cm/sec. The vapor pressure is related to specific humidity for a given atmospheric pressure, taken here as 1013 mb. Thus,

$$\frac{E}{\rho \, (q_w - q_a^o)} = 7.23 \times 10^{-3} \, u_2 \, x_o^{-.1} \tag{A34}$$

In order to construct the non-dimensional vapor flux on the left hand side of the above equation, the velocity at two

meters, u_2, must be converted to friction velocity, u_*. If, as a first approximation, it is assumed that the velocity profile is logarithmic (neutral conditions), the substitution of the velocity at two meters into equation (A34) yields for total average vapor flux:

$$W = [1.8 \times 10^{-2} \, z_o^{-1} \ln (\frac{200}{z_o})] \, x_o^{-.1} \qquad \text{(A35)}$$

where both W and x_o are dimensionless but not z_o. If the roughness length, z_o, is known, the coefficient in the brackets can be calculated. On the other hand, from tables 1 and 2 it can be seen that the exponent in the Harbeck formula corresponds to A_* and B_* both between 0 and -0.001. Therefore, the coefficient should have a value between .167 and .210. Solving for z_o in the coefficient in equation (A35) above yields z_o = .14 cm for the coefficient, a, equal to .167 and z_o = .05 cm for a = .210. Hence, an exponent of -0.1 seems to imply a roughness length approximately equal to 0.1 cm. However, this value is almost certainly too large, since the comparision is made assuming a logarithmic velocity profile. For an unstable velocity profile, which the non zero values of A_* and B_* require, the coefficient would be larger and the roughness length derived therefrom would be smaller. This is in agreement with the earlier findings of Brutsaert and Yeh (1970), who obtained a roughness length of z_o = .0213 cm by equating coefficients in the Harbeck and Sutton solutions.

The present results can readily be applied for engineering purposes. If the surface temperature and specific humidity are known upwind and downwind of a discontinuity and the upwind shear stress or wind profile were known, A_* and B_* can be determined. Then, from Tables 1 and 2 values of a and n may be determined or extrapolated. Equation (A30) should then yield an estimate for the average vapor flux over a given fetch, x_o, greater than, say, 10^4. For longer periods, such as one week or more, when the atmosphere may be assumed to te near neutral on the average, it is probably permissible to calculate lake evaporation by means of Sutton's solution or its empirical equivalent, the Harbeck formula.

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